Summer School 2026

Topics in Banach Space Theory

Various approaches to hyperconvexity in partial metric spaces

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Abstract

Partial metric spaces introduced by Matthews in 1994 play an important role among spaces endowed with non-Hausdorff topologies. Matthews showed, roughly speaking, how metric-like tools can be applied in such spaces, and he also indicated some applications of this class of spaces in the study of the denotational semantics of programming languages. Further applications of partial metrics can also be found in the geometry of Banach spaces.

On the other hand, the notion of hyperconvexity for metric spaces was introduced by Aronszajn and Panitchpakdi in 1956 in their study of extensions of uniformly continuous mappings between metric spaces. From a topological perspective, a hyperconvex space is an absolute retract via a nonexpansive retraction. By the theorem of Nachbin and Kelley, hyperconvex real Banach spaces can be treated as Stonian spaces $C(K)$ of all real-valued continuous functions on extremally disconnected compact Hausdorff spaces K.

In this talk, we present several approaches to defining hyperconvexity in partial metric spaces. In particular, we show that the analogue of the Aronszajn-Panitchpakdi notion of hyperconvexity fails to possess certain key properties that hold in the classical metric setting. Finally, we will discuss the problem of how to properly define Lipschitz continuity for mappings in a partial metric space and formulate some open problems related to these notions.

The talk is mainly based on the join paper with P. Kasprzak and O. Olela-Otafudu, Hyperconvexity in partial metric spaces: challenges and outlooks. arXiv preprint arXiv:2601.02279 (2026).