Summer School 2026

Topics in Banach Space Theory

Subideals of $\mathcal{L}(X)$ of higher order

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Abstract

Let X be a Banach space, and let $\mathcal{L}(X)$ denote the Banach algebra of bounded linear operators on X. Let $\mathcal{I} \subset \mathcal{L}(X)$ be a subalgebra and $n \in \mathbb{N}$. We say that $\mathcal{I}$ is an *algebraic n-subideal* of $\mathcal{L}(X)$ if there are subalgebras $\mathcal{I}_0, \ldots, \mathcal{I}_n$ of $\mathcal{L}(X)$ such that

$$\mathcal{I} = \mathcal{I}_n \subset \cdots \subset \mathcal{I}_1 \subset \mathcal{I}_0 = \mathcal{L}(X), \tag{1}$$

where $\mathcal{I}_k$ is an ideal of $\mathcal{I}_{k-1}$ for each $k = 1, \ldots, n$. If $\mathcal{I}$ and the intermediate ideals $\mathcal{I}_k$ in (1) are closed, we say that $\mathcal{I}$ is a *closed n-subideal* of $\mathcal{L}(X)$. We call an n-subideal $\mathcal{I}$ *non-trivial* if it is not an $(n-1)$-subideal of $\mathcal{L}(X)$. Examples of non-trivial algebraic 2-subideals of $\mathcal{L}(\ell_2)$ were obtained by Fong and Radjavi (1983) and later by Patnaik and Weiss. However, much less is known about subideals of order $n \geq 3$.

In this talk I will present a decreasing sequence $(\mathcal{I}_n)_{n=1}^{\infty}$, where each $\mathcal{I}_n$ is a non-trivial closed n-subideal of $\mathcal{L}(X)$ for certain direct sums X of distinct ℓ_p-spaces. Moreover, for classical Banach spaces X with an unconditional basis, the algebra $\mathcal{L}(X)$ contains decreasing sequences $(\mathcal{I}_n)_{n=1}^{\infty}$ contained in the ideal of compact operators, where each $\mathcal{I}_n$ is a non-trivial algebraic n-subideal of $\mathcal{L}(X)$.