Summer School 2026

Topics in Banach Space Theory

The two-disjoint-copies property for compact spaces, homogeneity, and connection with C_p-theory

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Day and time	Monday: 12:20–12:55

Abstract

A Tychonoff space X has the two-disjoint-copies property (2DCP) if there exists a sequence $(K_n)_n$ of non-empty compact subsets of X such that each K_n contains two disjoint subsets homeomorphic to K_{n+1}. Banakh, Kąkol and Śliwa showed that 2DCP for X yields an infinite-dimensional metrizable quotient of $C_p(X)$, while it is still a long-standing open question whether $C_p(X)$ has such a quotient for any infinite compact space X. The above concept as well as the last problem are closely related to Efimov's problem that has remained open for 40 years. We will discuss a number of conditions that imply 2DCP. For example, every locally homogeneous compact space, every space containing a copy of $\beta\omega$ or 2^ω has 2DCP although compact h-homogeneous spaces with 2DCP without such copies exist in ZFC. We prove that no scattered compact space has 2DCP but there exist in ZFC compact perfect spaces without 2DCP. This implies that for compact metric spaces X, the 2DCP is equivalent to the uncountability of X. We give positive classes among zero-dimensional compact spaces; for example, the Brech, as well as the Sobota-Zdomskyy compact spaces of Efimov type have 2DCP. Open questions are included.