Summer School 2026

Topics in Banach Space Theory

Countable tensor products and similarity to contraction semigroups

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Abstract

A bounded operator A on a Hilbert space H is said to be *similar to a contraction* if there exists a bounded invertible operator P on H such that $\|PAP^{-1}\| \leq 1$. This notion has been extensively studied, with fundamental contributions by G. Pisier, R. Paulsen, and others. In the setting of C_0-semigroups, however, similarity phenomena become considerably more delicate. A classical example due to P. R. Chernoff shows that a C_0-semigroup $\mathcal{T} := (T(t))_{t\geq 0}$ need not be similar to a contraction semigroup, even if each individual operator $T(t)$, $t > 0$, is itself similar to a contraction.

In this talk, based on recent joint work with Y. Tomilov, we discuss the problem of similarity to contraction semigroups without imposing any additional regularity assumptions. In our most general framework, given a countable index set I, we study countable tensor products of semigroups $\bigotimes_{\ell\in I} \mathcal{T}_\ell$ acting on complete and incomplete tensor products of Hilbert spaces $\bigotimes_{\ell\in I} H_\ell$. These constructions, introduced by von Neumann, extend the usual finite tensor product of Hilbert spaces.

Our main result characterizes when the tensor product $\bigotimes_{\ell\in I} \mathcal{T}_\ell$ is similar to a contraction semigroup in terms of the corresponding similarity properties of the factors $(\mathcal{T}_\ell)_{\ell\in I}$, together with a suitable balancing condition. As an application of this characterization, we construct new counterexamples concerning similarity to contraction semigroups which are, in a sense, optimal. In particular, we obtain a nilpotent immediately compact semigroup which is not similar to any contraction semigroup.