Summer School 2026

Topics in Banach Space Theory

Fixed point theory for Lipschitz operators in Banach spaces

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Day and time	Tuesday: 10:00–10:40 Wednesday: 11:15–11:55 Friday: 10:00–10:40

Abstract

Let C be a set and $T : C \to C$ be a mapping. A point $x \in C$ is said to be a fixed point for T when $T(x) = x$. As it may be expected, the existence of a fixed point is not fulfilled in general and it is going to depend on the features of the two main players of this game: the domain and the operator. During this course, the domain C will be a closed convex bounded subset of a Banach space X and T a Lipschitz operator, that is, there exists some $L > 0$ such that

$$d(Tx, Ty) \leq Ld(x, y), \quad \forall x, y \in C.$$

Two cases can be analyzed right way:

- If $L < 1$: we have Banach Contraction Principle to turn to and there is always a unique fixed point.
- If $L > 1$: we will expose the failure of the existence of a fixed point in very regular and smooth conditions, namely, the closed unit ball of a Hilbert space.

However, when $L = 1$ we face a world of possibilities, ranging from classic examples where the existence of a fixed point fails, to sophisticated arguments depending on the geometry of the Banach space that ensure the existence of fixed points.

During this course, we will try to understand some of the geometrical properties lying behind the scenes that allow us to prove the existence of some fixed points for 1-Lipschitz operators, as well as relate them to renorming theory and some other areas of research within the scope of Banach space theory.