Summer School 2026

Topics in Banach Space Theory

Lineability of smooth bumps not satisfying Rolle's theorem

Name	Miguel García Bravo
University	Universidad Complutense de Madrid
Day and time	Thursday: 13:10–13:35

Abstract

The main purpose of this talk is to prove the following result.

Theorem: *For many Banach spaces E such as $c_0(\Gamma)$ or $\ell_p(\Gamma)$, $1 < p < \infty$, where Γ is an infinite set, and given any open set $U \subset E$, the set of smooth bumps without critical points on their open support U, that is*

$$V = \{b \in \mathrm{Diff}(E; [0, \infty)) \colon b \textit{ is a bump with } U = \{x \in E : b(x) > 0\} \textit{ and } b'(x) \neq 0 \textit{ for all } x \in U\},$$

is τ-lineable where τ is the cardinality of any Hamel basis of E^*. This means that there exists an infinite dimensional subspace $W \subset V \cup \{0\}$ with cardinality τ.

The classical Rolle theorem ensures that for every continuous bump $b : \mathbb{R}^n \to [0, \infty)$ that is differentiable on $U = \{x \in \mathbb{R}^n : b(x) > 0\}$ there exists $x_0 \in U$ so that $f'(x_0) = 0$. Contrary to what happens in finite dimensions, Rolle's theorem fails in infinite dimensions. This was originally proved by Shkarin in 1990 for superreflexive Banach spaces. Since then, the class of Banach spaces for which Rolle's theorem fails has been substantially enlarged, and now includes, in particular, all infinite-dimensional Banach spaces admitting either a (not necessarily equivalent) Fréchet differentiable norm.

The theorem stated above shows that for many Banach spaces E for which the Rolle's theorem fails, there exist not only one bump function on V, but indeed an algebraically large amount of counterexamples.

This is a joint work with Mar Jiménez-Sevilla.