Summer School 2026

Topics in Banach Space Theory

Some Banach spaces with a 2-rotund norm

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Abstract

Let X be a Banach space. We say that a norm $\|\cdot\|$ on X is 2-rotund ($2R$) (resp. weakly 2-rotund ($W2R$)) if for every $(x_n) \subset X$ such that $\|x_n\| \leq 1$ ($n \geq 1$) and

$$\lim_{m,n\to\infty} \|x_m + x_n\| = 2,$$

there exists $x \in X$ such that $x = \lim_{n\to\infty} x_n$ strongly (resp. weakly). It follows from a characterization of reflexivity due to James that if X admits an equivalent $W2R$ norm then X is reflexive.

Hájek and Johanis proved the converse. i.e., every reflexive Banach space admits an equivalent $W2R$ norm, while Odell and Schlumprecht proved that every separable reflexive Banach space X admits an equivalent $2R$ norm.

However, it is an open question whether every (nonseparable) reflexive Banach space admits an equivalent $2R$ norm.

We consider the existence of $2R$ norms for certain families of nonseparable reflexive spaces, including Banach spaces with a symmetric or a subsymmetric basis, Baernstein spaces, abstract interpolation spaces, and tensor products. We also show that reflexive Banach spaces with an uncondtional basis admit a 1-unconditional $2R$ renorming and embed as a 1-complemented subspace of a Banach space with a 1-symmetric basis and a $2R$ norm.