Summer School 2026

Topics in Banach Space Theory

(λ^+)-injective Banach spaces

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Day and time	Monday: 11:35–12:10

Abstract

Injective Banach spaces form a classical meeting point of Banach-space geometry, extension theory, and the theory of L_∞-spaces. A natural quantitative refinement asks how the notions of λ-injectivity and (λ^+)-injectivity differ. A forgotten result of Pełczyński asserts that, for every $\lambda > 1$, these two notions are genuinely distinct: there exists a Banach space which is (λ^+)-injective but not λ-injective.

In this talk I will discuss a proof completing this result in the remaining range $\lambda > 2$. The key ingredient is a zero-sum construction $\Sigma_N(Y) \subset Z_\infty^N$, which amplifies relative projection constants by the factor $2 - 2/N$ while preserving the relevant non-attainment property. Iterating this construction reduces the problem to the previously known range $(1, 2]$, and hence establishes Pełczyński's theorem for all $\lambda > 1$. I will also explain how the same ideas yield a related Banach–Mazur distance estimate, including the bound

$$\operatorname{dist}\bigl(L_\infty[0,1], \ell_\infty\bigr) \leq 9 + 6\sqrt{3}.$$